647510 **BSCmaMJ204x-2C** **Seat No :_________**

B.Sc. Semester - 2 (NEP-2020) Examination
JUNE – 2024
MATH: Partial Derivatives & Differential Geometry (Major-4)

Time: 2:00 Hours **Marks: 50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one. (05)

(1) If u is homogeneous function of x and y with degree n then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$. Also if $z = f(u)$ then deduce that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$.

(2) If $u = sin^{-1}(x - y),\ x = 3t, y = 4t^3$ then show that $\frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$.

Que.1(B) Answer any one. (05)

(1) If $u = log(tanx + tany + tanz)$ then prove that $sin2x\frac{\partial u}{\partial x} + sin2y\frac{\partial u}{\partial y} + sin2z\frac{\partial u}{\partial z} = 2$.

(2) Define: Continuity of function of two variables.

If $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

then discuss the continuity of f at $(0,0)$.

Que.2(A) Answer any one. (05)

(1) State and prove Taylor's theorem for function of two variables.

(2) Find maxima and minima of the function $f(x,y) = xy + 27\left(\frac{1}{x} + \frac{1}{y}\right)$.

Que.2(B) Answer any one. (05)

(1) If $u = x^2 + y^2 + z^2, v = xy + yz + zx, w = x^3 + y^3 + z^3 - 3xyz$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$.

(2) If there is 1 % error in the measurement of half of major and minor axis of an ellipse then evaluate relative error in measure of its area.

Que.3(A) Answer any one. (05)

(1) Define: Radius of curvature. Prove that the radius of curvature at any point (x,y) on the curve $y = f(x)$ is $\rho = \frac{(1+{y_1}^2)^{\frac{3}{2}}}{y_2}$.

(2) Find radius of curvature at origin for the curve $x = a(\theta + sin\theta)$, $y = a(1 - cos\theta)$.

Que.3(B) Answer any one. (05)

(1) Show that radius of curvature at any point on the cardioid $r = a(1 - cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$.

(2) Find radius of curvature at origin for the curve $x^3 - 2x^2y + 3xy^2 - 4y^3 + 5x^2 - 6xy + 7y^2 - 8y = 0$.

Que.4(A) Answer any one. (05)

(1) Find the interval of values of x for which the curve $x^2y = a^2(x-y)$ is concave upwards or downwards. Find also the points of inflexion in each case.

(2) If $y = mx + c$ is an asymptote to the curve $y = f(x)$ then prove that $m = \lim_{x\to\pm\infty}\left(\frac{y}{x}\right)$ and $c = \lim_{x\to\pm\infty}(y - mx)$.

Que.4(B) Answer any one. (05)

(1) Find multiple points on the curve

$$x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$$ and determine its type.

(2) Find all the asymptotes of the curve $x^3 + y^3 - 3axy = 0$.

Que.5(A) Answer any one. (05)

(1) Verify Euler's theorem for the function $u = (ax + by)^{\frac{1}{3}}$.

(2) Expand $e^x cosy$ in powers of x and y up to degree three.

Que.5(B) Answer any one. (05)

(1) Find radius of curvature at any point (x, y) on the curve $y^2 = 4ax$.

(2) Define: (i) Point of inflexion (ii) Double point (iii) Cusp (iv) Node (v) Isolated point.

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